Analysis and Synthesis of Interconnected Systems: Application to Tuned Vibration Absorber Design for a Flexible Beam

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1 Introduction

Since the last decade, substantial research has been devoted to the analysis and synthesis of interconnected systems by addressing techniques from robust systems theory. An important result is the convex reformulation of the distributed controller design problem, for which the controller has the same interconnection scheme as the plant and the controller subunits having the same order as the ones on the plant [1]. However, this controller type is not always applicable. In some cases, it is desired to synthesize controllers with a much lower degree or with a free interconnection topology. In this work, two strategies are explored to solve structured distributed control problems. These are applied on the design of tuned vibration absorbers for a flexible beam.

2 Analysis and Synthesis of Interconnected Systems

The lower part of Figure 1 illustrates an interconnected system with equal subunits G. This system can be addressed as an uncertain one with the time and spatial frequency as uncertainties. Using techniques from robust systems analysis, one obtains an LMI condition, guaranteeing an input-output energy attenuation γ in a given time frequency interval:

$$\begin{bmatrix} I \\ G \end{bmatrix}^T \Theta(P_t, P_s, \gamma) \begin{bmatrix} I \\ G \end{bmatrix} \prec 0 \tag{1}$$

Θ is a Hermitian matrix which depends linear on respectively the time and spatial related multiplier P_t, P_s and on γ. In order to retrieve the best attenuation in a desired frequency range, appropriate distributed controllers are designed. In this work, the structure of the controller is fixed and by solving problem (1) while minimizing γ, the optimal control parameters are determined. Because G depends on those parameters, the problem is non-convex and currently two approaches are investigated to solve it effectively. As the non-convex matrix inequality can be decomposed in a convex and concave part, it can be solved by sequential convex programming [2]. Secondly, the problem can also be solved as a parametric program. The variables P_t, P_s and γ are parametrized as B-Splines which depend on the control parameters. The problem becomes convex and results in the best performance as a function of the control parameters.

3 Tuned Vibration Control for a Flexible Beam

The synthesis methods are applied on the design of periodically attached tuned vibration absorbers for a flexible beam in order to create a desired frequency stop band which reduces the noise radiation in this frequency range. Because the absorbers are attached periodically, the structure can be modelled as an interconnection of equal subunits (Figure 1), from which the system representation is derived from the finite element model of the beam. The input of the system is a vector containing all nodal forces acting on the beam, the outputs are the nodal displacements. Therefore γ expresses the worst-case compliance seen over all possible spatial force distributions. In order to create a desired frequency stop band, the optimal parameters for the tuned absorber are determined which minimize γ in this frequency interval. A first result is illustrated in Figure 2. Using parametric programming, the worst-case compliance γ in the given frequency range is determined as a function of the damping coefficient of the tuned absorber, for which the mass is fixed and the resonance frequency is tuned in the middle of the desired stop band.

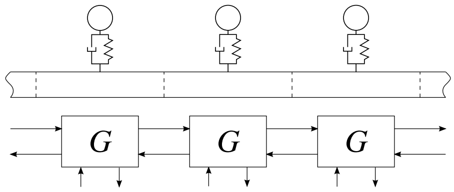

Figure 1: Beam with periodically attached tuned vibration absorbers modelled as an interconnected system.

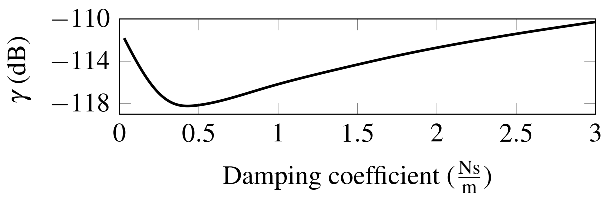

Figure 2: Worst-case compliance γ in a frequency band of 10 rad/s around 1000 rad/s for a beam with periodically attached tuned vibration absorbers with variable damping.

References

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